Engines powered by frequent measurements: work from system-bath correlations

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Abstract

Frequent unread measurements are shown to enable the extraction of work from changes in the system-bath *quantum-correlation* energy, which constitute a hitherto unexploited work resource. It becomes available only when the cycle is shorter than the non-Markovian bath-memory time.

In any engine, the working medium (henceforth–the system) exchanges energy with the outside world, which consists of baths and external forces. Part of the energy can be exploited as *useful work*, and the rest dissipates as thermal energy (heat). Although this division is not always clear-cut, once work and heat are identified [1], the standard (Kelvin) formulation of the second law [2] rules that *no work can be performed by the system in a single-bath engine over a cycle.*

An exception to this rule is that an observer acting as "Maxwell's demon" can commute information acquired by measuring the system into work in a single-bath engine[3]. Here we show that frequent measurements of the energy of a quantum system immersed in a *single* bath enable the system to do work in a cycle even if the measurement results are unread (or unknown). This finding cannot be attributed to "Maxwell's demon" operation[3–7], since unread quantum measurements provide no information. Nor can it be ascribed to quantum *coherence in the system*, which is the source of work in recently explored quantum heat engines (QHEs)[8, 9], since the unread measurements do not induce such coherence[10–12]. Instead, frequent unread measurements are shown to enable the extraction of work from changes in the system-bath *quantum-correlation* energy [10–14], which constitute a hitherto unexploited work resource. It becomes available only when the cycle is shorter than the non-Markovian (bath-memory) time.

Can we reconcile the second law with our findings? We recall that non-selective, brief quantum non-demolition (QND) measurements have been theoretically [10, 11, 13, 14] and experimentally[12] demonstrated to cause anomalous (oscillatory) behavior of temperature and entropy in a two-level system (TLS) coupled to a thermal bath, as their equilibrium is disturbed on non-Markovian time scales. These anomalies, similarly to the present results, suggest the need to reformulate the second law in the non-Markovian, quantum regime, wherein *the thermodynamic paradigm of system-bath partition* (separability)[1, 2, 15] may fail. Here we pursue this reformulation in the context of single-bath engines by revisiting the standard theory of open quantum systems (OQS) [2, 16] that underlies thermodynamics. This theory reduces the (otherwise unsolvable) system (S) + bath (B) dynamics to the effective *autonomous* evolution of the system, by means of its reduced state ρ_S. The standard OQS theory [2] assumes that under weak system-bath coupling the Born approximation holds, wherein the influence of the system on the bath is neglected, and the total state is the uncorrelated product state $\rho_{SB} \approx \rho_S \otimes \rho_B$, currently used in quantum thermodynamics

[1, 2, 15].

Here we show that the autonomous system evolution *seemingly* contradicts the second law, even in the weak-coupling regime: It allows a single-bath engine to extract work in a cycle, without any *apparent* cost. However, this perpetual-motion ("perpetuum mobile") paradox [17] is dispelled when we realize that the standard product-state assumption of OQS theory fails to account for the energy cost of a *brief* QND measurement of S, which *decorrelates* S and B, and thereby triggers their nonequlibrium dynamics. The remarkable aspect of this effect is that *useful work* is extractable from the measurement-induced S-B correlation-energy change only if the cycle is completed within a *non-Markovian* (bath-memory) time-scale.

The model Our model consists of a spin-1/2 system (S) with energy ω_a that is coupled via the spin-boson Hamiltonian H_{SB} [16] to a single thermal bath (B) of oscillators with inverse temperature $\beta = 1/T$ governed by Hamiltonian H_B. In addition, it interacts continuously with a "piston" (P). We assume that the piston is off-resonant and classical, so that it parametrically modulates the system Hamiltonian, which then acquires a time-dependent, e.g., sinusoidal, form: $H_S(t) = \omega(t)|e\rangle\langle e|$, where $\omega(t) = \omega_a + \Delta Sin\Omega t$ and $|e\rangle$ is the excited state of the TLS. More rigorous quantum-mechanical description of the piston and its coupling to the TLS can be given[18], without changing the main findings. At $t = t_m$ the system energy is *briefly* measured in a QND fashion. At $t_{end} = t_m + t_{cycle}$ the piston excitation is compared to that at t_m, so as to determine whether it did work on S or vice versa.

Apparent paradox of autonomous evolution The existing treatments of heat engines are based on the OQS theory assumption that the system evolution is autonomous (described by a Lindblad-type master equation for $\rho_S(t)$) and suffices for a thermodynamic analysis of an engine[1, 2, 15, 19]. Under this standard assumption, the following expression for the work over a cycle, W, is expected to hold [1]:

$$W = \oint tr\{\rho_S \dot{H}_S\} dt = \oint s(t)\dot{\omega}(t) dt. \tag{1}$$

Here $\omega(t)$ is the level-separation (frequency) of the piston-driven TLS, $s(t)$ is the polarization (population difference) of its energy-states $|e\rangle$, $|g\rangle$ and the cyclic integral is over a closed trajectory in the frequency-polarization plane. The work throughout a cycle is positive for

a clockwise trajectory and negative otherwise. The convention is that the work is negative if it is performed by the system on the piston [1]. According to Lindblad's H-theorem[2], which is the standard OQS expression of the second law, for a single bath engine $W \geq 0$, i.e., only the piston can do work on the system. Yet, strikingly, our high-precision numerical simulations and analytical results show (Fig. 1) that while the system interacts with a bath on non-Markovian time scales, it can close a counterclockwise trajectory, i.e., *net work can be performed in a cycle by the system on the piston ($W < 0$).*

The time-dependent non-Markovian master equation [10, 11, 16] reproduces (analytically) the results of the simulations. It yields the evolution of the TLS state, $\rho_S(t)$, that is initially a Gibbs state and remains diagonal in the energy basis, with parametrically time-dependent energy levels $E_e(t) - E_g(t) = \omega(t)$:

$$\dot{\rho}_{ee}(t) = R_g(t)\rho_{gg} - R_e(t)\rho_{ee}; \quad \dot{\rho}_{ee}(t) = -\dot{\rho}_{gg}(t) \tag{2}$$

Let us first assume adiabatic and Markovian conditions: i) $R_{g(e)}(t) \geq 0$; ii) adiabatic Gibbs-like stationary state that keeps the detailed thermal balance with temperature $k_B T = \frac{1}{\beta}$ at all times: $R_e(t)\rho_{ee}^{eq}(t) = R_g(t)\rho_{gg}^{eq}(t)$, $\rho_{jj}^{eq}(t) = Z^{-1}(t)\exp\{-\beta E_j(t)\}$, $j \in (g, e)$, $Z(t)$ being the normalization constant. Under these conditions , we can rigorously prove (Suppl. S1) that, since the entropies and energies of ρ_S are equal at the beginning and the end of a closed cycle, $W = -Q \geq 0$, where Q is the heat generation. This is the standard statement of the second law: a single-bath engine cannot yield work.

By contrast, in the strongly non-Markovian limit of fast modulation $\Omega \gg \Delta \gtrsim \omega_a$ the sign of W *oscillates* with Ω, for a fixed ω_a (Suppl. S2), i.e., Eq. (2) allows for *either positive or negative work*, as opposed to the Markovian limit (Suppl. S1). At low temperature and for weak but fast modulation, $\frac{\Delta}{\Omega} \ll 1$, the work is expressed in terms of the T-dependent bath-response $G_T(\omega)$ as (Suppl. S2)

$$W \approx \frac{\Delta}{2\pi}\int_{-\infty}^{\infty} G_T(\omega) \times \frac{2\pi}{(\omega^+)^2}\left(Sinc(\frac{2\pi}{\Omega}(\omega^+ + \Omega)) + Sinc(\frac{2\pi}{\Omega}(\omega^+ - \Omega))\right) d\omega \tag{3}$$

where $\omega^+ = \omega + \omega_a$. The sign of W *alternates* with the sinc argument, i.e., can be *negative.* Yet what is the source of energy for this work? The brief QND measurement at $t = t_m$ *does*

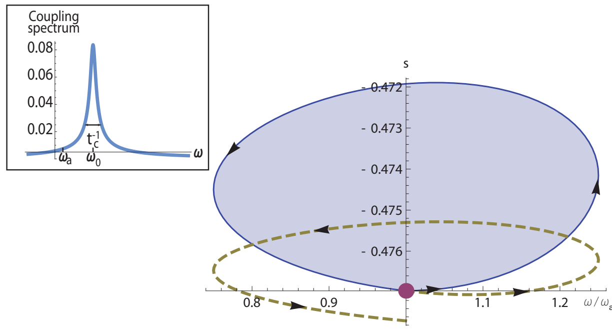

FIG. 1. Work extraction by the system in a single-bath engine. Main panel: Work extraction from a single-bath (B) engine by a TLS (S) whose polarization (s) is plotted vs. ω: results of high-precision simulations (see text).The cycles are driven by a piston-induced modulation of the unperturbed TLS energy, $\omega(t) = \omega_a + \Delta Sin\Omega t$. The measurement leaves the the system and the bath in a product state. The evolution direction is shown by the arrows. The counterclockwise sense implies that the *system performs work.* Two consecutive periods of the modulation are shown. While the first period starts with a measurement, the second does not. In the first period (solid blue curve) work is extracted from a single-bath engine and the polarization is also cyclic, the curve is closed and the enclosed area is the work $W < 0$ according to Eq. (1). In the second period (dashed brown curve) work is extracted ($W < 0$) as well, but the polarization is not cyclic, i.e., the curve is not closed, even though the modulation is periodic and the initial polarization (marked by a red point) is the same as in the first period. The curves are different in each period, since the initial supersystem (S+B) states are different, so that correlations with the bath are different (see text). The parameters for the solid and thin curves are $\hbar = k_B = 1$, $\omega_a = 1$, $\Delta = 1/4$, $\Omega = 5/2$. Inset: The S-B coupling spectrum, chosen to be a Lorentzian centered at ω_0 of width t_c^{-1} (inverse correlation or memory time of the bath response).The system-bath parameters are (in same units): $\omega_0 - \omega_a = 3/7$ where ω_a is the TLS resonance (transition) frequency unperturbed by the piston, $t_c = 10$ and the inverse bath temperature is $\beta = 3.74$.

not change the state of the system, ρ_S, nor that of the bath, ρ_B. Under the tenets of the standard OQS theory $\rho_{SB} \approx \rho_S \otimes \rho_B$, which implies that the measurement should carry *no cost* and have no bearing on W. Hence, the results of Eqs. (1)-(3), that work is done by S on P in a cycle *appear* to allow a perpetual-motion (*perpetuum-mobile)* engine[17].

Measurement cost To resolve the paradox, we must examine whether the assumption that underlies Lindblad's H-theorem, i.e., that S is autonomous, does apply on non-Markovian time scales. The indication that this standard assumption may fail in the non-Markovian domain is the drastic difference between the (precisely simulated) solid-line and dashed-curve evolution during the first and second periods of the modulation, respectively in Fig. 1. Namely, starting with the *same* ρ_S, i.e., at the same point in the $\omega - s$ plane, at the outset of the first and second modulation periods, *does not entail the same subsequent evolution* of $\rho_S(t)$. This implies that contrary to the standard assumption[1, 2] it is wrong

to assume that the system dynamics is always autonomous: in the non-Markovian domain, the correlations of the system with the bath are crucial!

Accordingly, the second law is upheld or reinstated (in the sense that perpetual motion becomes forbidden) only when we account for the energy and entropy cost of changing the "supersystem" state ρ_{SB} from its initial correlated form at equilibrium ($\rho_{SB}^{eq} = Z_S^{-1} e^{-\beta(H_S+H_B+H_{SB})} \neq \rho_S \otimes \rho_B$) to its post-measurement form $\rho_{SB}(t_m)$, (which is nearly a product state, as shown in Suppl. S3). This measurement cost comes about from changing the mean system-bath correlation energy from a negative equilibrium value [10, 11], $\langle H_{SB}(t = t_m^-)\rangle_{eq}$, to zero after the measurement, $\langle H_{SB}(t = t_m^+)\rangle = 0$, where $t = t_m^-$ and $t = t_m^+$ are the instants just before and just after the measurement, respectively. The energy injected to the "supersystem" by the measurement is $\Delta E = -\langle H_{SB}(t_m^-)\rangle_{eq}$, which is a *positive* quantity [10, 11]. Eq. (2) *cannot* account for this cost, which requires the analysis of ρ_{SB}.

This cost becomes evident in a description of the evolution in terms of the total Hamiltonian H_{tot} and the corresponding state ρ_{tot} that encompass the degrees of freedom of the *detector and the supersystem*, yielding the work

$$W_{tot} = \oint tr\{\rho_{tot}\dot{H}_{tot}\}dt. \tag{4}$$

It is important that *a "true" cycle* of ρ_{tot} in Eq. (4) *may be very different from the "pseudo-cycle"* in Eq. (1) which only ensures that ρ_S return to its initial value after a modulation period (Fig. 1) The non-negativity of the work $W_{tot} \geq 0$, under a *cyclic unitary* evolution of the Hamiltonian (H_{tot}), encompassing the supersystem and the detector and starting from a brief disturbance of the equilibrium can be proven (Suppl. S4) completely generally . For a cycle of duration τ

$$W_{tot}(\tau) = Tr\left(U(\tau)\rho_{tot}(0)U^\dagger(\tau)H_{tot}(0) - \rho_{tot}(0)H_{tot}(0)\right). \tag{5}$$

where the first term is the final mean energy and the second is the initial one. Because the total dynamics is unitary, the entropy of ρ_{tot} is fixed. This implies that the final mean energy (first term) must be greater than or equal to the initial one (second term), as the thermal-

equilibrium initial state minimizes the mean energy at fixed entropy. Hence $W_{tot}(\tau) \geq 0.$, i.e., (modulation) Eq. (4) may still allow *the system* to do net work on the piston during the cycle (W<0), but it compensates for this work by the energy cost ΔE of the non-selective QND measurement, so that rigorously, $W_{tot} = W + \Delta E \geq 0$ during the cycle, in keeping with the second law. As we show numerically (Fig. 2) part of the energy stored in $\langle H_{SB}(t)\rangle$ (after the measurement) is converted into work. The strong dependence of the sign of W on the cycle duration t_{cycle} proves that work retrieval from $\langle H_{SB}\rangle$ is limited to non-Markovian time scales (Fig. 2a–Inset): t_{cycle} should be much shorter than t_c, the bath memory time, to ensure work performance by the system.

The maximal work that can be done by the system on the piston via an unread (*non-selective*) measurement, as described above, is given by[20] $(W_{non-sel})_{Max} = -\Delta E + T\Delta\mathscr{S}$, ΔE being the energy invested by the detector in the measurement, T the bath temperature and $\Delta\mathscr{S}$ the entropy change of ρ_{SB} caused by the measurement[21]. The ratio of this work to ΔE is a measure of the maximal efficiency or reversibility, i.e., the ability of the system to recover the invested energy as useful work from the correlations with the bath at equilibrium

$$\epsilon_{Max} = \frac{-(W_{non-sel})_{Max}}{\Delta E} = 1 - \frac{T\Delta\mathscr{S}(\rho_{SB})}{-(\langle H_{SB}\rangle)_{eq}} \leq 1 \tag{6}$$

where $\Delta E = -(\langle H_{SB}\rangle)_{eq} > 0$ and $\Delta\mathscr{S}(\rho_{SB}) > 0$.

No need for resetting the detector It might be expected that resetting (purifying) the detector is necessary if we wish to reuse it and that would add to the thermodynamic cost [4, 5] of multiple cycles, i.e., reduce the extractable work after one cycle. However, the detector (D) can be repeatedly used *without resetting*, no matter how mixed its state, since its information *is unread*. Nevertheless, it performs a *genuine* measuring in that it becomes *correlated* with S. To avoid the adverse effects of such correlations, we add after the fast modulation a relaxation period during which *the correlations between S and D* decay owing to the single-bath. Thus, after the fast modulation we wait long enough, so that the S-B and D density matrices *decorrelate via thermal relaxation* and revert to a product state: $\rho_{S+B+D} \to \rho_{SB} \otimes \rho_D$. After this relaxation period (Fig. 2b) the detector can be reused in the next cycle and have the *same effect* as in the first, i.e., again erase the system-bath correlations. Hence, there is no need to reset the detector for further use.

Discussion The proposed work anomalies cannot be explained by viewing the unread

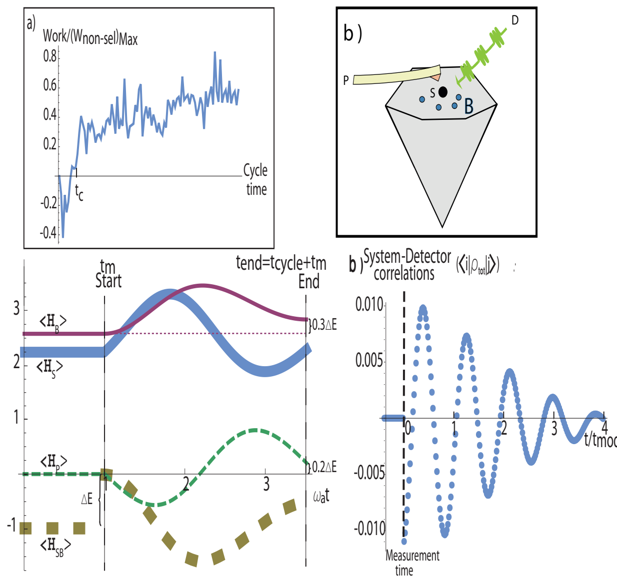

FIG. 2. Simulations of work extraction in a single-bath engine. Main panel: Simulation of the first-cycle evolution of the energy of the system $\langle H_S \rangle$ (solid thick blue line), the bath $\langle H_B \rangle$ (solid thin purple line), the classical piston ($\langle H_P \rangle$) (dashed thin green line), and the system-bath correlations $\langle H_{SB} \rangle$ (dashed thick brown line), same parameters as in Fig. 1. The cycle starts with a measurement at time $\omega_a t = 1$ and ends at $\omega_a t = 3.51$. The measurement invests $\Delta E = -\langle H_{SB}(t = t_m) \rangle$ in the system. $\langle H_P \rangle$ has gained energy during the cycle, but not at the expense of $\langle H_S \rangle$ which returns to its initial value. Since $\langle H_B \rangle$ has also gained energy, the source of work is $\langle H_{SB} \rangle$, the system-bath (correlation-dependent) interaction. The simulations imply that $W_{tot} = W + \Delta E \geq 0$, in accordance with Eq. (4), although $W = \langle H_P(t_m) \rangle - \langle H_P(t_{end}) \rangle < 0$. (b) Decay of the off-diagonal system-detector (S-D) elements (correlations) $\langle i|\rho_{tot}|j \rangle$ under H_{tot} (Eq. (2)) where $|i\rangle = |e, 1, n\rangle$ and $|j\rangle = |g, 0, n \pm 1\rangle$, the entries denoting the system, detector and bath quantum numbers, respectively. The detector frequency is 10/7. The decay time of the correlations is that of the oscillations envelope, here ~ 4 modulation periods ($4t_{mod}$). After this time the detector can be reused for the next cycle. Inset: (a): work done by the piston on the system W (normalized by $(W_{non-sel})_{Max}$, the maximal work obtainable via a non-selective measurement (Eq.(6)) as a function of the cycle duration t_{cycle}. It is seen that $W < 0$ (work done by the system) requires $t_{cycle} \lesssim t_c$, t_c being the bath memory time. (b): NV-center spin (S) in diamond coupled to a nuclear spin-bath(B) is driven by a cantilever that can act as a piston (P). The detector is a periodic spin readout that acts on S.

detector as a fictitious additional "bath", since the detector continuously equilibrates with the single bath. The second law is upheld only if we account for the change of *correlations* between the system and the *single bath*, caused by the brief measurement (Fig. 2, Main Panel): This change, the crux of the present results, allows a single-bath engine to do work that is unaccounted for by Lindblad's H-theorem[2] and prompts a reformulation of the second law for open quantum systems that undergo cycles in the non-Markovian time

domain.

Unlike cases[19] where *the master equation is simply not valid*, here the non-Markovian master equation[10–12, 22] adequately describes the system. What fails is the standard premise underlying Lindblad's H-theorem that the system dynamics is autonomous. This failure may have far-reaching implications, since the H-theorem is routinely used as a test of the validity or physicality of open-system analysis[19]. Instead, in the non-Markovian time domain, we have shown that the analysis must be extended to the *total* (combined) state of the detector, the system and the bath. Only such an analysis can rid us of the perpetual-motion paradox and yet allow for work by the system on the piston, as opposed to Markovian analysis[21]. This opens the door to a revision of traditional thermodynamics that would extend it beyond the system-bath partition (separability) paradigm, so as to allow for system-bath correlations.

On the practical side, the present engine model, in which the system is always coupled to a single bath and yet may perform useful work, is potentially important for nanoscale systems totally embedded in a single bath, so that conventional thermodynamic cycles may be impossible to implement. Nanomechanics[23] and spintronics[24] provide a variety of testbeds for the predicted measurement-based QHE: In particular, *mechanical work* is obtainable in setups where the "piston" is a nanomechanical cantilever that is magnetically coupled to a nitrogen-vacancy (NV) electron spin (qubit) in diamond [25] that interacts with a nuclear spin bath (Fig. 2-Inset b). The cycles in Fig. 1,2 are realizable in this setup by QND measurements[26] (readout) of the qubit. The long memory time of the spin bath, $t_c \gg \mu sec$ and the ability to monitor single spins are appealing features. It is also interesting to study cases where the resource that can power such an engine may be "gratuitious", i.e., natural processes that decorrelate the system from the bath at controlled intervals less than t_c. The performance of such an engine may be optimized by selecting the measurement rate that appropriately steers the non-Markovian dynamics[22, 27, 28]. Further investigation may include brief disturbances other than measurements, e.g., phase flips of a TLS in a spin bath[28].

To conclude, the present findings lead us to reformulate the second law for quantum scenarios wherein the thermodynamic paradigm of system-bath separability fails, as follows: *A quantum system can perform work in a closed cycle in a single-bath engine, through changes in the system-bath correlations within the memory time of the bath.* Most saliently,

work can be even extracted in the presence of a zero-temperature bath via a measurement.

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S1 No work can be extracted from a single Markovian bath engine in a closed cycle

To prove our result consider the entropy production rate in the Markov approximation [2]

$$\begin{gathered}\sigma \equiv -\sum_j \dot{\rho}_{jj}(\ln\rho_{jj} - \ln\rho_{jj}^{eq}) = \\ -(R_g\rho_{gg} - R_e\rho_{ee})\ln\frac{\rho_{ee}}{\rho_{ee}^{eq}} + (R_g\rho_{gg} - R_e\rho_{ee})\ln\frac{\rho_{gg}}{\rho_{gg}^{eq}} = \\ -R_g\rho_{gg}^{eq}(xlny - xlnx + x - y) - \\ R_e\rho_{ee}^{eq}(ylnx - ylny + y - x) \geq 0\end{gathered} \tag{S1}$$

where $x = \frac{\rho_{gg}}{\rho_{gg}^{eq}}$ and $y = \frac{\rho_{ee}}{\rho_{ee}^{eq}}$. Notice, that $R_g\rho_{gg}^{eq}(x-y) + R_e\rho_{ee}^{eq}(y-x) = 0$ due to adiabaticity. The inequality in (S1) is obtained from the relation $a\ln a - a\ln b + b - a \geq 0$ (for $a, b \geq 0$) and assumption A). It implies the following inequality for the entropy $S(t) =$

$-k_B \sum_j \rho_{jj}(t) \ln \rho_{jj}(t)$.

$$\dot{S} = -\sum_j \dot{\rho}_{jj} \ln \rho_{jj} \geq -\sum_j \dot{\rho}_{jj} \ln \rho_{jj}^{eq} = \frac{1}{T}\dot{Q}$$
$$\dot{Q} = \sum_j \dot{\rho}_{jj} E_j \ . \qquad \text{(S2)}$$

where we used the fact that $\sum_j \dot{\rho}_{jj} \ln Z(t) = \ln Z(t) \frac{d}{dt} \sum_j \rho_{jj} = 0$.

Since in a closed cycle the entropies and internal energies in the initial and final states of the system are equal, $W = -Q \geq 0$ (which is the second law of thermodynamics). This means that we cannot extract work from a single Markovian bath engine.

S2 Analytical work calculation in the non-Markovian regime We wish to evaluate the work performed by a qubit in contact with a bath at temperature T after a non-selective measurement of its energy, over a period (cycle) of its energy modulation of the form $\omega(t) = \omega_a + \Delta Sin\Omega t$. This requires the evaluation of [1]

$$W^{ext}_{cycle} = -\int_0^{\frac{2\pi}{\Omega}} s(t)\dot{\omega} dt. \qquad \text{(S3)}$$

We shall use the results of the weak-coupling non-Markovian master equation[12, 22], whereby

$$s(t) = e^{-J(t)}(\int \Delta R(t')e^{J(t')}dt' + s(0)). \qquad \text{(S4)}$$

Here $J = J_g + J_e$, $J_{g(e)} = \int_0^t R_{g(e)}(t')dt'$ and $\Delta R = \frac{1}{2}(R_g(t) - R_e(t))$, depend on the non-Markovian bath-induced transition rates $R_e(t)$ ($|e\rangle \mapsto |g\rangle$) and $R_g(t)$ ($|g\rangle \mapsto |e\rangle$). Both J and R are proportional to the square of the system-bath coupling strength, η. Since the coupling is assumed weak, $s(t)$ can be expanded as $s(t) \approx (1 - J(t))s(0) + \Delta J(t)$. The first term does not depend on the time and does not contribute to the work. Plugging the rest in the work definition and rearranging

$$W = \int_0^{\frac{2\pi}{\Omega}} \frac{J_g(t)J_e(\frac{2\pi}{\Omega}) - J_e(t)J_g(\frac{2\pi}{\Omega})}{J(\frac{2\pi}{\Omega})} \Omega\Delta Cos(\Omega t)dt \qquad \text{(S5)}$$

where

$$\int_0^{\frac{2\pi}{\Omega}} J_{g(e)}(t)\Omega\Delta Cos\Omega t dt =$$
$$\Omega\Delta \int_{-\infty}^{\infty} d\omega G_T(\omega)\times$$
$$\sum_{nm} \frac{J_n(-\frac{\Delta}{\Omega})J_m(-\frac{\Delta}{\Omega})}{(\omega^{\pm}+n\Omega)(\omega^{\pm}+m\Omega)}$$
$$\Bigg(\frac{Sin(\frac{2\pi}{\Omega}(\omega^{\pm}+(n+1)\Omega+\frac{\pi}{2}(n-m))-Sin(\frac{\pi}{2}(n-m))}{\omega^{\pm}+(n+1)\Omega}$$
$$+\frac{Sin(\frac{2\pi}{\Omega}(\omega^{\pm}+(n-1)\Omega+\frac{\pi}{2}(n-m))-Sin(\frac{\pi}{2}(n-m))}{\omega^{\pm}+(n-1)\Omega}\Bigg) \tag{S6}$$

where $\omega^{\pm}=\omega\pm\omega_a$This expression allows for *either positive or negative work extraction*, as opposed to the Markovian limit.

S3 The measurement process

The time-dependent system-detector coupling has the form

$$H_{SD}(t) = h(t)\left|e\right\rangle\left\langle e\right|(\left|0\right\rangle\left\langle 0\right| + \left|1\right\rangle\left\langle 1\right| - \left|0\right\rangle\left\langle 1\right| - \left|1\right\rangle\left\langle 0\right|) \tag{S7}$$

where

$$h(t) = \frac{\pi}{4\tau_m}\left(tanh^2\left(\frac{t-t_m}{\tau_m}\right)-1\right) \tag{S8}$$

is a smooth temporal profile of the system coupling to the detector qubits during the measurement at time t_m which lasts over time τ_m.
This form of the single-measurement Hamiltonian H_{SD} is chosen so that

$$e^{-i\int_0^{\tau_m} dt\frac{H_{SD}(t)}{\hbar}} = U_C \tag{S9}$$

where U_C denotes to the CNOT operation (with the detector qubit, as the target qubit). If the measurement duration τ_m is much shorter than the other time scales, then only H_{SD} is non-negligible during this time, and the entire action is well approximated by the CNOT operator U_C. This becomes exact in the impulsive limit $\tau_m \to 0$.
The measurement outcomes are averaged over (for nonselective measurements), by tracing

out the detector degrees of freedom. As long as the detector state is of the form

$$\rho_D = 1/2I + a\sigma_z \tag{S10}$$

where a is real, the measurement will not affect ρ_S, which is diagonal in the energy basis:

$$\begin{gathered}\rho_S \longmapsto Tr_D\left(U_C\rho_S \otimes \rho_D U_C^{\dagger}\right) = \\ \left|e\right\rangle\left\langle e\right|\rho_S\left|e\right\rangle\left\langle e\right| + \left|g\right\rangle\left\langle g\right|\rho_S\left|g\right\rangle\left\langle g\right|\end{gathered} \tag{S11}$$

i.e., the diagonal elements of ρ_S are *unchanged*, and the off-diagonal elements are erased. Since the system is entangled with the bath, the effect of the measurement on ρ_{SB} is:

$$\begin{gathered}\rho_{SB} \longmapsto Tr_D\left(U_C\rho_{SB} \otimes \rho_D U_C^{\dagger}\right) = \\ \left|e\right\rangle\left\langle e\right|\rho_{SB}\left|e\right\rangle\left\langle e\right| + \left|g\right\rangle\left\langle g\right|\rho_{SB}\left|g\right\rangle\left\langle g\right| \equiv \\ \rho_{ee}^{B}\left|e\right\rangle\left\langle e\right| + \rho_{gg}^{B}\left|g\right\rangle\left\langle g\right|\end{gathered} \tag{S12}$$

where ρ_{ee}^{B} and ρ_{gg}^{B} are bath states correlated to $|e\rangle$ and $|g\rangle$ respectively. Thus, the post-measurement ρ_{SB} is block-diagonal in the energy states of the system. It can be shown[10, 11] to be close to a product state of ρ_S and ρ_B.

S4 Non-negativity of work in a cycle The total Hamiltonian is $H_{tot} = H_0 + H_D + H_{BD} + H_{SD}$, H_{SD} is the system-detector coupling term, the bath-detector coupling H_{BD} allows a fast decorrelation between the system and detector (see below), and H_0 describes the "supersystem", system+bath. Using the fact that the total Hamiltonian is cyclic, $H_{tot}(\tau) = H_{tot}(0)$, the total work in a cycle can be written as

$$\begin{gathered}W_{tot}(\tau) = \int_0^{\tau} Tr\left(U(t)\rho_{tot}(0)U^{\dagger}(t)\right)\dot{H}_{tot}(t))dt = \\ Tr\left(\rho_{tot}(\tau)H_{tot}(\tau) - \rho_{tot}(0)H_{tot}(0)\right) - \\ \int_0^{\tau} Tr\left(\dot{\rho}_{tot}(t)H_{tot}(t)\right)dt\end{gathered} \tag{S13}$$

Upon inserting the expression for $\dot{\rho}_{tot}(t)$ and calculating the trace we find that the second term on the RHS is zero. The first term thus represents the energy change of the supersystem. Since initially the supersystem and the detector were at thermal equilibrium, any energy change should be *positive*. The second term on the RHS being zero, we then obtain Eq. (5).